GROUPS OF ORDERS 49-63

SUMMARY

order	49	50	51	52	53	54	55	56	57	58	59	60	61	62	63
# abelian	2	2	1	2	1	3	1	3	1	1	1	2	1	1	2
# other decomposable	0	1	0	1	0	6	0	4	0	0	0	8	0	0	1
# other indecomp	0	2	0	2	0	6	1	6	1	1	0	3	0	1	1
TOTAL	**2**	**5**	**1**	**5**	**1**	**15**	**2**	**13**	**2**	**2**	**1**	**13**	**1**	**2**	**4**

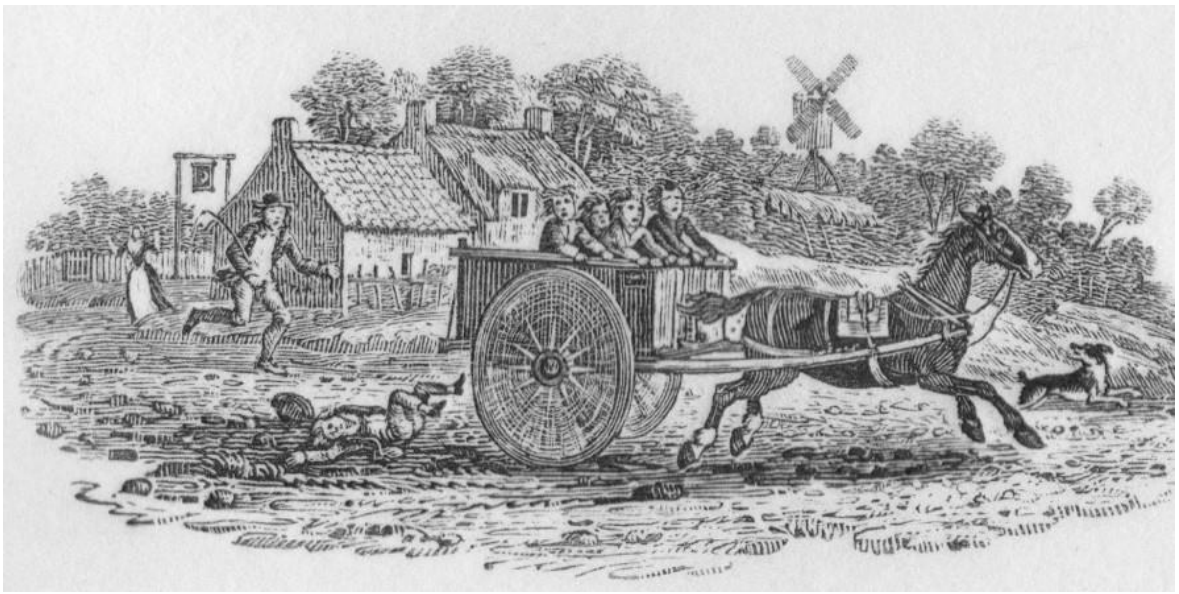
$49 = \langle A^{49} \rangle$

$49 = 7^2 = \langle A^7, B^7 \rangle$

$50 = \langle A^{50} \rangle = 25 \times 2$

$50.2 = 25 \times 2 = \langle A^{25}, B^2 \rangle$

$50.3 = D_{10} \times 5 = \langle A^5, B^2, C^5, BA = A^{-1}B \rangle$



50.4 = ⟨A^5, B^5, C^2, $CA = A^{-1}C$, $CB = B^{-1}C$⟩

CC	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**	**13**	**14**
elts	1	A A^4	AB A^4B^4	AB^2 A^4B^3	AB^3 A^4B^2	AB^4 A^4B	A^2 A^3	A^2B A^3B^4	A^2B^2 A^3B^3	A^2B^3 A^3B^2	A^2B^4 A^3B	B B^4	B^2 B^3	A^mB^nC

C	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**	**13**	**14**	
#	**1**	**2**	**2**	**2**	**2**	**2**	**2**	**2**	**2**	**2**	**2**	**2**	**2**	**25**	ℋ
	1	1	1	1	1	1	1	1	1	1	1	1	1	1	G
	1	1	1	1	1	1	1	1	1	1	1	1	1	–1	1
	2	2	c_1	c_2	c_2	c_1	2	c_1	c_2	c_2	c_1	c_1	c_2	0	2
	2	2	c_2	c_1	c_1	c_2	2	c_2	c_1	c_1	c_2	c_2	c_1	0	2
	2	c_1	c_1	c_1	c_1	c_1	c_2	c_2	c_2	c_2	c_2	2	2	0	3
	2	c_1	c_2	c_1	c_2	2	c_2	c_2	c_1	2	c_1	c_1	c_2	0	4
	2	c_1	c_1	2	c_2	c_2	c_2	c_1	c_1	c_2	2	c_2	c_1	0	5
	2	c_2	c_2	c_2	c_2	c_2	c_1	c_1	c_1	c_1	c_1	2	2	0	3
	2	c_2	c_2	c_1	2	c_1	c_1	2	c_1	c_2	c_2	c_1	c_2	0	6
	2	c_2	c_1	c_1	c_2	2	c_1	c_1	c_2	2	c_2	c_2	c_1	0	4
	2	c_1	c_1	c_1	c_1	c_1	c_2	c_2	c_2	c_2	c_2	2	2	0	3
	2	c_2	c_2	c_2	c_2	c_2	c_1	c_1	c_1	c_1	c_1	2	2	0	3
	2	2	c_1	c_2	c_2	c_1	2	c_1	c_2	c_2	c_1	c_1	c_2	0	2
	2	2	c_2	c_1	c_1	c_2	2	c_2	c_1	c_1	c_2	c_2	c_1	0	2
∧	**1**	**5**	**5**	**5**	**5**	**5**	**5**	**5**	**5**	**5**	**5**	**5**	**5**	**2**	

$c_k = 2\cos(2k\pi/5)$

	Classes	**H**	**G/H**	ℋ	
1	1 + 2 + 3 + 4 + 5 + 6 + 7 + 8 + 9 + 10 + 11 + 12 + 13	$\mathbf{5^2}$	**2**	1	**G′**
2	1 + 2 + 7	**5**	$\mathbf{D_{10}}$	2	
3	1 + 12 + 13	**5**	$\mathbf{D_{10}}$	3	
4	1 + 6 + 10	**5**	$\mathbf{D_{10}}$	4	
5	1 + 4 + 11	**5**	$\mathbf{D_{10}}$	5	
6	1 + 5 + 8	**5**	$\mathbf{D_{10}}$	6	

Z = ℜ = Φ = 1

Sylow subgroups: [**2**] × 25, [$\mathbf{5^2}$] **Maximal subgroups:** [$\mathbf{5^2}$], [$\mathbf{D_{10}}$] × 24

50.5 = D_{50} = $\langle A^{25}, B^2, BA = A^{-1}B \rangle$

CC	1	2	3	4	5	6	7	8	9	10	11	12	13	14
elts	1	A A^{24}	A^2 A^{23}	A^3 A^{22}	A^4 A^{21}	A^5 A^{20}	A^6 A^{19}	A^7 A^{18}	A^8 A^{17}	A^9 A^{16}	A^{10} A^{15}	A^{11} A^{14}	A^{12} A^{13}	A^nB

C	1	2	3	4	5	6	7	8	9	10	11	12	13	14	
#	1	2	2	2	2	2	2	2	2	2	2	2	2	25	ℵ
1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	G
1	1	1	1	1	1	1	1	1	1	1	1	1	1	–1	1
2	c_1	c_2	c_3	c_4	c_5	c_6	c_7	c_8	c_9	c_{10}	c_{11}	c_{12}	0	0	
2	c_2	c_4	c_6	c_8	c_{10}	c_{12}	c_{11}	c_9	c_7	c_5	c_3	c_1	0	0	
2	c_3	c_6	c_9	c_{12}	c_{10}	c_7	c_4	c_1	c_2	c_5	c_8	c_{11}	0	0	
2	c_4	c_8	c_{12}	c_9	c_5	c_1	c_3	c_7	c_{11}	c_{10}	c_6	c_2	0	0	
2	c_5	c_{10}	c_{10}	c_5	2	c_5	c_{10}	c_{10}	c_5	2	c_5	c_{10}	0	2	
2	c_6	c_{12}	c_7	c_1	c_5	c_{11}	c_9	c_2	c_4	c_{10}	c_9	c_3	0	0	
2	c_7	c_{11}	c_4	c_3	c_{10}	c_9	c_1	c_6	c_{12}	c_5	c_2	c_9	0	0	
2	c_8	c_9	c_1	c_7	c_{10}	c_2	c_6	c_{11}	c_3	c_5	c_{12}	c_4	0	0	
2	c_9	c_7	c_2	c_{11}	c_5	c_4	c_{12}	c_3	c_6	c_{10}	c_1	c_8	0	0	
2	c_{10}	c_5	c_5	c_{10}	2	c_{10}	c_5	c_5	c_{10}	2	c_{10}	c_5	0	2	
2	c_{11}	c_3	c_8	c_6	c_5	c_9	c_2	c_{12}	c_1	c_{10}	c_4	c_7	0	0	
2	c_{12}	c_1	c_{11}	c_2	c_{10}	c_3	c_9	c_4	c_8	c_5	c_7	c_6	0	0	
^	1	25	25	25	25	25	25	25	25	25	25	25	25	2	

$c_k = 2\cos(2k\pi/25)$

	Classes	H	G/H	ℵ	
1	1 + 2 + 3 + 4 + 5 + 6 + 7 + 8 + 9 + 10 + 11 + 12 + 13	**25**	**2**	1	**G′**
2	1 + 6 + 11	**5**	**D_{10}**	2	

Z = **N** = **Φ** = [**1**]
Sylow subgroups: [**2**] × 25, [**25**] **Maximal subgroups:** [**25**], [**D_{10}**] × 5

51 = $\langle A^{51} \rangle$ = 17 × 3

52 = $\langle A^{52} \rangle$ = 13 × 4

52.2 = 26 × 2 = $\langle A^{13}, B^2, C^2 \rangle$

52.3 = D_{26} × 2 = $\langle A^{13}, B^2, C^2, BA = A^{-1}B \rangle$

52.4 = D_{52} = $\langle A^{26}, B^2, BA = A^{-1}B \rangle$

CC	Γ_1	Γ_2	Γ_3	Γ_4	Γ_5	Γ_6	Γ_7	Γ_8	Γ_9	Γ_{10}	Γ_{11}	Γ_{12}	Γ_{13}	Γ_{14}	Γ_{15}	Γ_{16}
elts	1	A^{13}	A A^{25}	A^2 A^{24}	A^3 A^{23}	A^4 A^{22}	A^5 A^{21}	A^6 A^{20}	A^7 A^{19}	A^8 A^{18}	A^9 A^{17}	A^{10} A^{16}	A^{11} A^{15}	A^{12} A^{14}	$A^{2n}B$	$A^{2n+1}B$

C	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**	**13**	**14**	**15**	**16**	
#	**1**	**1**	**2**	**2**	**2**	**2**	**2**	**2**	**2**	**2**	**2**	**2**	**2**	**2**	**13**	**13**	$\mathfrak{K}$
	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	G
	1	1	1	1	1	1	1	1	1	1	1	1	1	1	–1	–1	1
	1	–1	–1	1	–1	1	–1	1	–1	1	–1	1	–1	1	1	–1	2
	1	–1	–1	1	–1	1	–1	1	–1	1	–1	1	–1	1	–1	1	3
	2	2	c_1	c_2	c_3	c_4	c_5	c_6	c_7	c_8	c_9	c_{10}	c_{11}	c_{12}	0	0	4
	2	2	c_2	c_4	c_6	c_8	c_{10}	c_{12}	c_1	c_3	c_5	c_7	c_9	c_{11}	0	0	4
	2	2	c_3	c_6	c_9	c_{12}	c_2	c_5	c_8	c_{11}	c_1	c_4	c_7	c_{10}	0	0	4
	2	2	c_4	c_8	c_{12}	c_3	c_7	c_{11}	c_2	c_6	c_{10}	c_1	c_5	c_9	0	0	4
	2	2	c_5	c_{10}	c_2	c_7	c_{12}	c_4	c_9	c_1	c_6	c_{11}	c_3	c_8	0	0	4
	2	2	c_6	c_{12}	c_5	c_{11}	c_4	c_{10}	c_3	c_9	c_2	c_8	c_1	c_7	0	0	4
	2	2	c_7	c_1	c_8	c_2	c_9	c_3	c_{10}	c_4	c_{11}	c_5	c_{12}	c_6	0	0	4
	2	2	c_8	c_3	c_{11}	c_6	c_1	c_9	c_4	c_{12}	c_7	c_4	c_{10}	c_5	0	0	4
	2	2	c_9	c_5	c_1	c_{10}	c_6	c_2	c_{11}	c_7	c_3	c_{12}	c_8	c_4	0	0	4
	2	2	c_{10}	c_7	c_4	c_1	c_{11}	c_8	c_5	c_4	c_{12}	c_9	c_6	c_3	0	0	4
	2	2	c_{11}	c_9	c_7	c_5	c_3	c_1	c_{12}	c_{10}	c_8	c_6	c_4	c_2	0	0	4
	2	2	c_{12}	c_{11}	c_{10}	c_9	c_8	c_7	c_6	c_5	c_4	c_3	c_2	c_1	0	0	4
^	**1**	**2**	**26**	**13**	**26**	**13**	**26**	**13**	**26**	**13**	**26**	**13**	**26**	**13**	**2**	**2**	

	Classes	**H**	**G/H**	∩	
1	1 + 2 + 3 + 4 + 5 + 6 + 7 + 8 + 9 + 10 + 11 + 12 + 13 + 14	**26**	**2**	1	
2	1 + 4 + 6 + 8 + 10 + 12 + 14 + 15	**D_{26}**	**2**	2	
3	1 + 4 + 6 + 8 + 10 + 12 + 14 + 16	**D_{26}**	**2**	3	
4	1 + 4 + 6 + 8 + 10 + 12 + 14	**13**	**2^2**	2∩3	**G′**
5	1 + 2	**2**	**D_{26}**	4	**Z**

$\mathfrak{N} = \mathbf{\Phi} = [\mathbf{1}]$

Sylow subgroups: [$\mathbf{2^2}$] × 13, [**13**] **Maximal subgroups:** [**26**], [$\mathbf{D_{26}}$] × 2

52.5 = $M_{13,4}^{(5)}$ = ⟨A^{13}, B^4, $BA = A^5B$⟩

CC	1	2			3				4				5	6	7
#	1	A	A^5	A^8	A^{12}	A^2	A^3	A^{10}	A^{11}	A^4	A^6	A^7	A^9	A^nB^2	A^nB	A^nB^3

C	1	2	3	4	5	6	7	
#	1	4	4	4	13	13	13	ℵ
	1	1	1	1	1	1	1	G
	1	1	1	1	1	–1	–1	1
	1	1	1	1	–1	i	–i	2
	1	1	1	1	–1	–i	i	2
	4	$c_1 + c_5$	$c_2 + c_3$	$c_4 + c_6$	0	0	0	0
	4	$c_2 + c_3$	$c_4 + c_6$	$c_1 + c_5$	0	0	0	0
	4	$c_4 + c_6$	$c_1 + c_5$	$c_2 + c_3$	0	0	0	0
^	1	13	13	13	2	4	4	

	Classes	H	G/H	∩	
1	1 + 2 + 3 + 4 + 5	**26**	**2**	1	
2	1 + 2 + 3 + 4	**13**	**4**	2	**G′**

Z = 𝔄 = **Φ** = [**1**]
Sylow subgroups: [2^2] × 13, [**13**] **Maximal subgroups:** [D_{26}], [**4**] × 13

53 = ⟨A^{53}⟩

54 = ⟨A^{54}⟩ = 27 × 2

54.02 = 18 × 3 = ⟨A^{18}, B^3⟩

54.03 = 6 × 3^2 = ⟨A^6, B^3, C^3⟩

54.04 = 27.4 × 2 = ⟨A^3, B^3, C^3, D^2, $CB = A^2BC$⟩

54.05 = $M_{9,3}^{(4)}$ × 2 = ⟨A^9, B^3, C^2, $BA = A^4B$⟩

54.06 = D_6 × 3^2 = ⟨A^3, B^2, C^3, D^3, $BA = A^{-1}B$⟩

54.07 = D_6 × 9 = ⟨A^3, B^2, C^9, $BA = A^{-1}B$⟩

54.08 = 18.4 × 3 = ⟨A^3, B^3, C^2, D^3, $CA = A^{-1}C$, $CB = B^{-1}C$⟩

54.09 = D_{18} × 3 = ⟨A^9, B^2, C^3, $BA = A^{-1}B$⟩

54.10 = ⟨A^3, B^3, C^3, D^2, $CB = A^2BC$, $DB = B^{-1}D$, $DC = C^{-1}D$⟩

1	**2**	**3**	**4**	**5**	**6**	**7**
1	A	A^2	A^nB A^nB^2	A^nC A^nC^2	A^nBC $A^nB^2C^2$	A^nB^2C A^nBC^2

8	**9**	**10**
D BD B^2D CD C^2D BCD ABC^2D AB^2CD $A^2B^2C^2D$	AD ABD AB^2D ACD AC^2D ABCD A^2BC^2D A^2B^2CD B^2C^2D	A^2D A^2BD A^2B^2D A^2CD A^2C^2D A^2BCD BC^2D B^2CD AB^2C^2D

C	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	
#	**1**	**1**	**1**	**6**	**6**	**6**	**6**	**9**	**9**	**9**	ℵ
	1	1	1	1	1	1	1	1	1	1	G
	1	1	1	1	1	1	1	–1	–1	–1	1
	2	2	2	2	–1	–1	–1	0	0	0	2
	2	2	2	–1	2	–1	–1	0	0	0	3
	2	2	2	–1	–1	2	–1	0	0	0	4
	2	2	2	–1	–1	–1	2	0	0	0	5
	3	3ω	$3\omega^2$	0	0	0	0	1	ω	ω^2	0
	3	$3\omega^2$	3ω	0	0	0	0	1	ω^2	ω	0
	3	3ω	$3\omega^2$	0	0	0	0	–1	–ω	$-\omega^2$	0
	3	$3\omega^2$	3ω	0	0	0	0	–1	$-\omega^2$	–ω	0
^	**1**	**3**	**3**	**3**	**3**	**3**	**3**	**2**	**6**	**6**	

	Classes	**H**	**G/H**		
1	1 + 2 + 3 + 4 + 5 + 6 + 7	**27.4**	**3**	1	**G′**
2	1 + 2 + 3 + 4	$\mathbf{3^2}$	$\mathbf{D_6}$	2	
3	1 + 2 + 3 + 5	$\mathbf{3^2}$	$\mathbf{D_6}$	3	
4	1 + 2 + 3 + 6	$\mathbf{3^2}$	$\mathbf{D_6}$	4	
5	1 + 2 + 3 + 7	$\mathbf{3^2}$	$\mathbf{D_6}$	5	
6	1 + 2 + 3	**3**	**18.4**	2∩3	**Z = ℜ = Φ**

Sylow subgroups: [**2**] × 9, [**27.4**] **Maximal subgroups:** [**27.4**], [$\mathbf{D_6 \times 3}$] × 12

54.11 = ⟨A^3, B^3, C^3, D^2, $DA = A^{-1}D$, $CB = A^2BC$, $DB = B^{-1}D$⟩

1	**2**	**3**	**4**	**5**	**6**	**7**
1	A	C	C^2	B	BC	BC^2
	A^2	AC	AC^2	AB	ABC	ABC^2
		A^2C	A^2C^2	A^2B	A^2BC	A^2BC^2
				B^2	B^2C	B^2C^2
				AB^2	AB^2C	AB^2C^2
				A^2B^2	A^2B^2C	$A^2B^2C^2$

8			**9**			**10**		
D	BD	B^2D	CD	BCD	B^2CD	C^2D	BC^2D	B^2C^2D
AD	ABD	AB^2D	ACD	ABCD	AB^2CD	AC^2D	ABC^2D	AB^2C^2D
A^2D	A^2BD	A^2B^2D	A^2CD	A^2BCD	A^2B^2CD	A^2C^2D	A^2BC^2D	$A^2B^2C^2D$

C	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	
#	**1**	**2**	**3**	**3**	**6**	**6**	**6**	**9**	**9**	**9**	ℵ
	1	1	1	1	1	1	1	1	1	1	G
	1	1	1	1	1	1	1	–1	–1	–1	1
	1	1	ω	ω^2	1	ω	ω^2	–1	–ω	$-\omega^2$	2
	1	1	ω^2	ω	1	ω^2	ω	–1	$-\omega^2$	–ω	2
	1	1	ω	ω^2	1	ω	ω^2	1	ω	ω^2	3
	1	1	ω^2	ω	1	ω^2	ω	1	ω^2	ω	3
	2	2	2	2	–1	–1	–1	0	0	0	4
	2	2	2ω	$2\omega^2$	–1	–ω	$-\omega^2$	0	0	0	5
	2	2	$2\omega^2$	2ω	–1	$-\omega^2$	–ω	0	0	0	5
	6	–3	0	0	0	0	0	0	0	0	0
^	**1**	**3**	**3**	**3**	**3**	**3**	**3**	**2**	**6**	**6**	

Normal subgroups

	Classes	**H**	**G/H**	∩	
1	1+ 2 + 3 + 4 + 5 + 6 + 7	**27.4**	**2**	1	
2	1 + 2 + 5	**3^2**	**6**	2	**G′**
3	1 + 2 + 5 + 8	**18.4**	**3**	3	
4	1 + 2 + 3 + 4	**3^2**	**D_6**	4	
5	1 + 2	**3**	**$D_6 \times 3$**	5	

Sylow subgroups: [**2**] × 9, [**27.4**] **Maximal subgroups:** [**27.4**], [**$D_6 \times 3$**] × 3, [**18.4**]

54.12 = ⟨A^9, B^6, $BA = A^5B$⟩

1	2	3	4	5
1	A^3 A^6	A A^2 A^4 A^5 A^7 A^8	B^2 A^3B^2 A^6B^2	AB^2 A^2B^2 A^4B^2 A^5B^2 A^7B^2 A^8B^2

6	7	8	9	10
B^4 A^3B^4 A^6B^4	AB^4 A^2B^4 A^4B^4 A^5B^4 A^7B^4 A^8B^4	A^nB	A^nB^3	A^nB^5

C	1	2	3	4	5	6	7	8	9	10	
#	1	2	6	3	6	3	6	9	9	9	ℵ
	1	1	1	1	1	1	1	1	1	1	G
	1	1	1	1	1	1	1	–1	–1	–1	1
	2	2	–1	2	–1	2	–1	0	0	0	2
	1	1	1	ω^2	ω^2	ω	ω	ω	1	ω^2	3
	1	1	1	ω^2	ω^2	ω	ω	$-\omega$	–1	$-\omega^2$	4
	2	2	–1	$2\omega^2$	$-\omega^2$	2ω	$-\omega$	0	0	0	5
	1	1	1	ω	ω	ω^2	ω^2	ω^2	1	ω	3
	1	1	1	ω	ω	ω^2	ω^2	$-\omega^2$	–1	$-\omega$	4
	2	2	–1	2ω	$-\omega$	$2\omega^2$	$-\omega^2$	0	0	0	5
	6	–3	0	0	0	0	0	0	0	0	0
^	1	3	9	3	9	3	9	6	2	6	

	Classes	H	G/H	
1	1 + 2 + 3 + 4 + 5 + 6 + 7	**27.5**	**2**	1	
2	1 + 2 + 4 + 6	**3^2**	**D_6**	2	
3	1 + 2 + 3 + 9	**D_{18}**	**3**	3	
4	1 + 2 + 3	**9**	**6**	4	**G′**
5	1 + 2	**3**	**$D_6 \times 3$**	5	

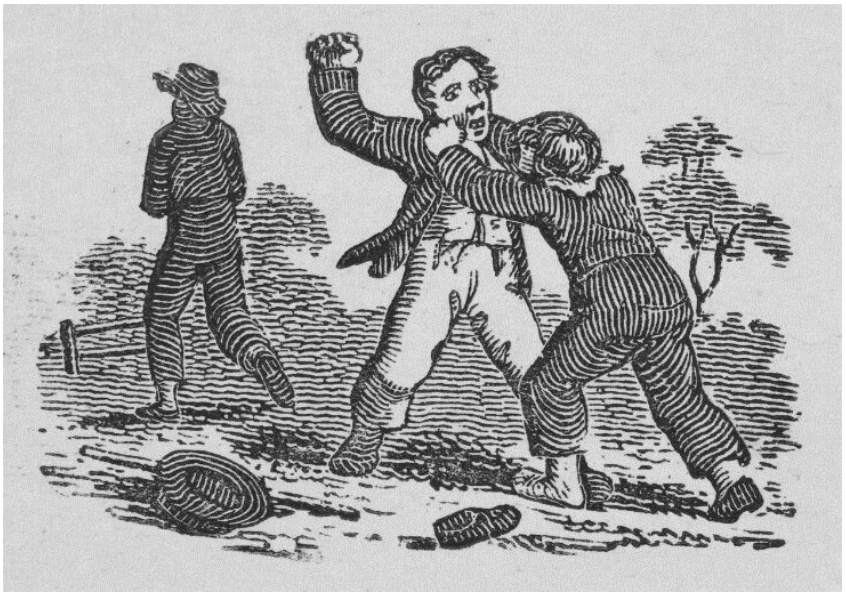
Sylow subgroups: [**2**] × 9, [**27.5**] **Maximal subgroups:** [$M_{9,3}^{(4)}$], [$D_6 \times 3$] × 3, [Q_8]

54.13 = ⟨A^3, B^3, C^3, D^2, DA = $A^{-1}D$, DB = $B^{-1}D$, DC = $C^{-1}D$⟩

1	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**
1	A A^2	B B^2	AB A^2B^2	A^2B AB^2	C C^2	AC A^2C^2	A^2C AC^2	BC B^2C^2	ABC $A^2B^2C^2$	A^2BC AB^2C^2

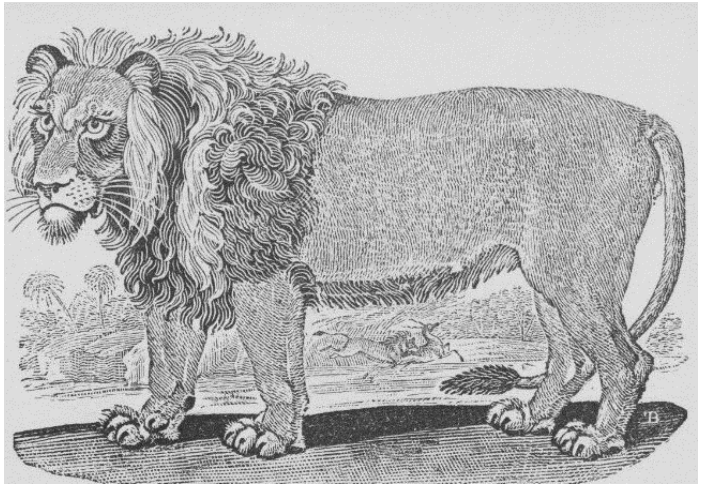
12	**13**	**14**	**15**
B^2C BC^2	AB^2C A^2BC^2	A^2B^2C ABC^2	$A^mB^nC^rD$

C	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**	**13**	**14**	**15**	
#	1	2	2	2	2	2	2	2	2	2	2	2	2	2	27	ℵ
	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	G
	1	1	1	1	1	1	1	1	1	1	1	1	1	1	–1	1
	2	2	2	2	2	–1	–1	–1	–1	–1	–1	–1	–1	–1	0	2
	2	2	–1	–1	–1	2	2	2	–1	–1	–1	–1	–1	–1	0	3
	2	2	–1	–1	–1	–1	–1	–1	2	2	2	–1	–1	–1	0	4
	2	2	–1	–1	–1	–1	–1	–1	–1	–1	–1	2	2	2	0	5
	2	–1	2	–1	–1	2	–1	–1	2	–1	–1	2	–1	–1	0	6
	2	–1	2	–1	–1	–1	2	–1	–1	2	–1	–1	2	–1	0	7
	2	–1	2	–1	–1	–1	–1	2	–1	–1	2	–1	–1	2	0	8
	2	–1	–1	2	–1	2	–1	–1	–1	2	–1	–1	–1	2	0	9
	2	–1	–1	2	–1	–1	2	–1	–1	–1	2	2	–1	–1	0	10
	2	–1	–1	2	–1	–1	–1	2	2	–1	–1	–1	2	–1	0	11
	2	–1	–1	–1	2	2	–1	–1	–1	–1	2	–1	2	–1	0	12
	2	–1	–1	–1	2	–1	2	–1	2	–1	–1	–1	–1	2	0	13
	2	–1	–1	–1	2	–1	–1	2	–1	2	–1	2	–1	–1	0	14
^	**1**	**3**	**3**	**3**	**3**	**3**	**3**	**3**	**3**	**3**	**3**	**3**	**3**	**3**	**2**	



	Classes	H	G/H	∩	
1	1 + 2 + 3 + 4 + 5 + 6 + 7 + 8 + 9 + 10 + 11 + 12 + 13 + 14	$\mathbf{3^3}$	**2**	1	**G′**
2	1 + 2 + 3 + 4 + 5	$\mathbf{3^2}$	$\mathbf{D_6}$	2	
3	1 + 2 + 6 + 7 + 8	$\mathbf{3^2}$	$\mathbf{D_6}$	3	
4	1 + 2 + 9 + 10 + 11	$\mathbf{3^2}$	$\mathbf{D_6}$	4	
5	1 + 2 + 12 + 13 + 14	$\mathbf{3^2}$	$\mathbf{D_6}$	5	
6	1 + 3 + 6 + 9 + 12	$\mathbf{3^2}$	$\mathbf{D_6}$	6	
7	1 + 3 + 7 + 10 + 13	$\mathbf{3^2}$	$\mathbf{D_6}$	7	
8	1 + 3 + 8 + 11 + 14	$\mathbf{3^2}$	$\mathbf{D_6}$	8	
9	1 + 4 + 6 + 10 + 14	$\mathbf{3^2}$	$\mathbf{D_6}$	9	
10	1 + 4 + 7 + 11 + 12	$\mathbf{3^2}$	$\mathbf{D_6}$	10	
11	1 + 4 + 8 + 9 + 13	$\mathbf{3^2}$	$\mathbf{D_6}$	11	
12	1 + 5 + 6 + 11 + 13	$\mathbf{3^2}$	$\mathbf{D_6}$	12	
13	1 + 5 + 7 + 9 + 14	$\mathbf{3^2}$	$\mathbf{D_6}$	13	
14	1 + 5 + 8 + 10 + 12	$\mathbf{3^2}$	$\mathbf{D_6}$	14	
15	1 + 2	**3**	**18.4**	2∩3	
16	1 + 3	**3**	**18.4**	2∩6	
17	1 + 4	**3**	**18.4**	2∩9	
18	1 + 5	**3**	**18.4**	2∩12	
19	1 + 6	**3**	**18.4**	3∩6	
20	1 + 7	**3**	**18.4**	3∩7	
21	1 + 8	**3**	**18.4**	3∩8	
22	1 + 9	**3**	**18.4**	4∩6	
23	1 + 10	**3**	**18.4**	4∩7	
24	1 + 11	**3**	**18.4**	4∩8	
25	1 + 12	**3**	**18.4**	5∩6	
26	1 + 13	**3**	**18.4**	5∩7	
27	1 + 14	**3**	**18.4**	5∩8	

Sylow subgroups: [**2**] × 27, [$\mathbf{3^2}$] **Maximal subgroups:** [$\mathbf{3^3}$], [**18.4**] × 39



54.14 = ⟨A^9, B^3, C^2, $CA = A^{-1}C$, $CB = B^{-1}C$⟩

1	**2**	**3**	**4**	**5**	**6**	**7**	**8**
1	A^3 A^6	B B^2	A^3B A^6B^2	A^6B A^3B^2	A A^8	A^2 A^7	A^4 A^5

9	**10**	**11**	**12**	**13**	**14**	**15**
AB A^8B^2	A^2B^2 A^7B	A^4B A^5B^2	AB^2 A^8B	A^2B A^7B^2	A^4B^2 A^5B	A^mB^nC

C	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**	**13**	**14**	**15**	
#	**1**	**2**	**2**	**2**	**2**	**2**	**2**	**2**	**2**	**2**	**2**	**2**	**2**	**2**	**27**	ℋ
	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	G
	1	1	1	1	1	1	1	1	1	1	1	1	1	1	–1	1
	2	2	2	2	2	–1	–1	–1	–1	–1	–1	–1	–1	–1	0	2
	2	2	–1	–1	–1	2	2	2	–1	–1	–1	–1	–1	–1	0	3
	2	2	–1	–1	–1	–1	–1	–1	2	2	2	–1	–1	–1	0	4
	2	2	–1	–1	–1	–1	–1	–1	–1	–1	–1	2	2	2	0	5
	2	–1	2	–1	–1	c_1	c_2	c_4	c_1	c_2	c_4	c_1	c_2	c_4	0	6
	2	–1	2	–1	–1	c_2	c_4	c_1	c_2	c_4	c_1	c_2	c_4	c_1	0	6
	2	–1	2	–1	–1	c_4	c_1	c_2	c_4	c_1	c_2	c_4	c_1	c_2	0	6
	2	–1	–1	2	–1	c_1	c_2	c_4	c_2	c_4	c_1	c_4	c_1	c_2	0	7
	2	–1	–1	2	–1	c_2	c_4	c_1	c_4	c_1	c_2	c_1	c_2	c_4	0	7
	2	–1	–1	2	–1	c_4	c_1	c_2	c_1	c_2	c_4	c_2	c_4	c_1	0	7
	2	–1	–1	–1	2	c_1	c_2	c_4	c_4	c_1	c_2	c_2	c_4	c_1	0	8
	2	–1	–1	–1	2	c_2	c_4	c_1	c_1	c_2	c_4	c_4	c_1	c_2	0	8
	2	–1	–1	–1	2	c_4	c_1	c_2	c_2	c_4	c_1	c_1	c_2	c_4	0	8
^	**1**	**3**	**3**	**3**	**3**	**9**	**9**	**9**	**9**	**9**	**9**	**9**	**9**	**9**	**2**	

$c_k = 2\cos(2k\pi/9)$

	Classes	**H**	**G/H**	∩
1	1 + 2 + 3 + 4 + 5 + 6 + 7 + 8 + 9 + 10 + 11 + 12 + 13 + 14	9 × 3	3	1
2	1 + 2 + 3 + 4 + 5	3^2	$\mathbf{D}_6$	2
3	1 + 2 + 6 + 7 + 8	9	6	3
4	1 + 2 + 9 + 10 + 11	9	$\mathbf{D}_6$	4
5	1 + 2 + 12 + 13 + 14	9	$\mathbf{D}_6$	5
6	1 + 3	3	$\mathbf{D}_{18}$	6
7	1 + 4	3	$\mathbf{D}_{18}$	7
8	1 + 5	3	$\mathbf{D}_{18}$	8
9	1 + 2	3	18.4	2∩3

Sylow subgroups: [**2**] × 27, [**9 × 3**] **Maximal subgroups:** [**9 × 3**], [$\mathbf{D}_{18}$] × 12

54.15 = D_{54} = ⟨A^{27}, B^2, $BA = A^{-1}B$⟩

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
1	A A^{26}	A^2 A^{25}	A^3 A^{24}	A^4 A^{23}	A^5 A^{22}	A^6 A^{21}	A^7 A^{20}	A^8 A^{19}	A^9 A^{18}	A^{10} A^{17}	A^{11} A^{16}	A^{12} A^{15}	A^{13} A^{14}	A^nB

C	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**	**13**	**14**	**15**	
#	**1**	**2**	**2**	**2**	**2**	**2**	**2**	**2**	**2**	**2**	**2**	**2**	**2**	**2**	**27**	$\mathcal{K}$
	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	G
	1	1	1	1	1	1	1	1	1	1	1	1	1	1	–1	1
	2	c_1	c_2	c_3	c_4	c_5	c_6	c_7	c_8	c_9	c_{10}	c_{11}	c_{12}	c_{13}	0	0
	2	c_2	c_4	c_6	c_8	c_{10}	c_{12}	c_{13}	c_{11}	c_9	c_7	c_5	c_3	c_1	0	0
	2	c_3	c_6	c_9	c_{12}	c_{12}	c_9	c_6	c_3	2	c_3	c_6	c_9	c_{12}	0	2
	2	c_4	c_8	c_{12}	c_{11}	c_7	c_3	c_1	c_5	c_9	c_{13}	c_{10}	c_8	c_2	0	0
	2	c_5	c_{10}	c_{12}	c_7	c_2	c_3	c_8	c_{13}	c_9	c_4	c_1	c_6	c_{11}	0	0
	2	c_6	c_{12}	c_9	c_3	c_3	c_9	c_{12}	c_6	2	c_6	c_{12}	c_9	c_3	0	2
	2	c_7	c_{13}	c_6	c_1	c_8	c_{12}	c_5	c_2	c_9	c_{11}	c_4	c_3	c_{10}	0	0
	2	c_8	c_{11}	c_3	c_5	c_{13}	c_6	c_2	c_{10}	c_9	c_1	c_7	c_{12}	c_4	0	0
	2	c_9	c_9	2	c_9	c_9	2	c_9	c_9	2	c_9	c_9	2	c_9	0	3
	2	c_{10}	c_7	c_3	c_{13}	c_4	c_6	c_{11}	c_1	c_9	c_8	c_2	c_{12}	c_5	0	0
	2	c_{11}	c_5	c_6	c_{10}	c_1	c_{12}	c_4	c_7	c_9	c_2	c_{13}	c_3	c_8	0	0
	2	c_{12}	c_3	c_9	c_8	c_6	c_9	c_3	c_{12}	2	c_{12}	c_3	c_9	c_6	0	2
	2	c_{13}	c_1	c_{12}	c_2	c_{11}	c_3	c_{10}	c_4	c_9	c_5	c_8	c_6	c_7	0	0
^	**1**	**27**	**27**	**9**	**27**	**27**	**9**	**27**	**27**	**9**	**27**	**27**	**9**	**27**	**2**	

$c_k = 2\cos(2k\pi/27)$

	Classes	**H**	**G/H**	∩	
1	1 + 2 + 3 + 4 + 5 + 6 + 7 + 8 + 9 + 10 + 11 + 12 + 13 + 14	**27**	**3**	1	**G′**
2	1 + 4 + 7 + 10 + 13	**9**	**D_6**	3	
3	1 + 10	**3**	**D_{18}**	2	

Sylow subgroups: [**2**] × 27, [**27**] **Maximal subgroups:** [**27**], [**D_{18}**] × 3

55 = ⟨A^{55}⟩ = 11 × 5

55.2 = $M_{11,5}^{(4)}$ = ⟨A^{11}, B^5, $BA = A^4B$⟩

1	2	3	4	5	6	7
1	A A^3 A^4 A^5 A^9	A^2 A^6 A^7 A^8 A^{10}	A^nB	A^nB^2	A^nB^3	A^nB^4

C	1	2	3	4	5	6	7	
#	1	5	5	11	11	11	11	ℵ
1	1	1	1	1	1	1	1	G
1	1	1	θ	θ^2	θ^3	θ^4	1
1	1	1	θ^2	θ^4	θ	θ^3	1
1	1	1	θ^3	θ	θ^4	θ^2	1
1	1	1	θ^4	θ^3	θ^2	θ	1
5	$\frac{-1+3\sqrt{11}i}{2}$	$\frac{-1-3\sqrt{11}i}{2}$	0	0	0	0	0
5	$\frac{-1-3\sqrt{11}i}{2}$	$\frac{-1+3\sqrt{11}i}{2}$	0	0	0	0	0
^	1	11	11	5	5	5	5	

	Classes	H	G/H	∩	
1	1 + 2 + 3	**11**	**5**	1	**G′**

Sylow and maximal subgroups: [**5**] × 11, [**11**]

56 = ⟨A^{56}⟩ = 8 × 7

56.02 = 28 × 2 = ⟨A^{28}, B^2⟩

56.03 = 14 × 2^2 = ⟨A^{14}, B^2, C^2⟩

56.04 = D_8 × 7 = ⟨A^4, B^2, C^7, $BA = A^{-1}B$⟩

56.05 = Q_8 × 7 = ⟨A^4, $B^2 = A^2$, C^7, $BA = A^{-1}B$⟩

56.06 = D_{14} × 2^2 = ⟨A^7, B^2, C^2, D^2, $BA = A^{-1}B$⟩

56.07 = D_{14} × 4 = ⟨A^7, B^2, C^4, $BA = A^{-1}B$⟩

56.08 = D_{56} = $\langle A^{28}, B^2, BA = A^{-1}B \rangle$

1	2	3	4	5	6	7	8	9
1	A^{14}	A^7 A^{21}	A A^{27}	A^2 A^{26}	A^3 A^{25}	A^4 A^{24}	A^5 A^{23}	A^6 A^{22}

10	11	12	13	14	15	16	17
A^8 A^{20}	A^9 A^{19}	A^{10} A^{18}	A^{11} A^{17}	A^{12} A^{16}	A^{13} A^{15}	$A^{2n}B$	$A^{2n+1}B$

C	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	
#	1	1	2	2	2	2	2	2	2	2	2	2	2	2	2	14	14	ℵ
	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	G
	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	–1	–1	1
	1	1	–1	–1	1	–1	1	–1	1	1	–1	1	–1	1	–1	1	–1	2
	1	1	–1	–1	1	–1	1	–1	1	1	–1	1	–1	1	–1	–1	1	3
	2	–2	0	0	–2	0	2	0	–2	2	0	–2	0	2	0	0	0	4
	2	–2	0	c_1	c_2	c_3	c_4	c_5	c_6	c_8	c_9	c_{10}	c_{11}	c_{12}	c_{13}	0	0	0
	2	2	–2	c_2	c_4	c_6	c_8	c_{10}	c_{12}	c_{12}	c_{10}	c_8	c_6	c_4	c_2	0	0	5
	2	–2	0	c_3	c_6	c_9	c_{12}	c_{13}	c_{10}	c_4	c_1	c_2	c_5	c_8	c_{11}	0	0	0
	2	2	2	c_4	c_8	c_{12}	c_{12}	c_8	c_4	c_4	c_8	c_{12}	c_{12}	c_8	c_4	0	0	6
	2	–2	0	c_5	c_{10}	c_{13}	c_8	c_3	c_2	c_{12}	c_{11}	c_6	c_1	c_4	c_9	0	0	0
	2	2	–2	c_6	c_{12}	c_{10}	c_4	c_2	c_8	c_8	c_2	c_4	c_{10}	c_{12}	c_6	0	0	5
	2	2	2	c_8	c_{12}	c_4	c_4	c_{12}	c_8	c_8	c_{12}	c_4	c_4	c_{12}	c_8	0	0	6
	2	–2	0	c_9	c_{10}	c_1	c_8	c_{11}	c_2	c_{12}	c_3	c_6	c_{13}	c_4	c_9	0	0	0
	2	2	–2	c_{10}	c_8	c_2	c_{12}	c_6	c_4	c_4	c_6	c_{12}	c_2	c_8	c_{10}	0	0	5
	2	–2	0	c_{11}	c_6	c_5	c_{12}	c_1	c_{10}	c_4	c_{13}	c_2	c_9	c_8	c_3	0	0	0
	2	2	2	c_{12}	c_4	c_8	c_8	c_4	c_{12}	c_{12}	c_4	c_8	c_8	c_4	c_{12}	0	0	6
	2	–2	0	c_{13}	c_2	c_{11}	c_4	c_9	c_6	c_8	c_9	c_{10}	c_3	c_{12}	c_1	0	0	0
^	1	2	4	28	14	28	14	28	14	14	28	14	28	14	28	2	2	

	Classes	H	G/H	ℵ	
1	1 + 2 + 3 + 5 + 6 + 7 + 8 + 9 + 10 + 11 + 12 + 13 + 14 + 15	**28**	**2**	1	
2	1 + 2 + 5 + 7 + 9 + 10 + 12 + 14 + 16	$\mathbf{D_{28}}$	**2**	2	
3	1 + 2 + 5 + 7 + 9 + 10 + 12 + 14 + 17	$\mathbf{D_{28}}$	**2**	3	
4	1 + 7 + 10 + 14	**7**	$\mathbf{D_{14}}$	4	
5	1 + 2	**2**	$\mathbf{D_{28}}$	5	**Z**
6	1 + 2 + 3	**4**	$\mathbf{D_8}$	6	
7	1 + 2 + 5 + 7 + 9 + 10 + 12 + 14	**14**	$\mathbf{2^2}$	1∩2	**G′**

Sylow subgroups: [**4 × 2**] × 7, [**7**] **Maximal subgroups:** [**14 × 2**], [$\mathbf{D_{7,4}}$] × 2, [**4 × 2**] × 7

56.09 = $D_{7,8}$ = $\langle A^7, B^8, BA = A^{-1}B \rangle$

1	2	3	4	5	6	7	8	9	10
1	B^4	B^2	B^6	A A^6	AB^4 A^6B^4	AB^2 A^6B^2	AB^6 A^6B^2	A^2 A^5	A^2B^4 A^5B^4

11	12	13	14	15	16	17	18	19	20
A^2B^2 A^5B^6	A^2B^6 A^5B^2	A^3 A^4	A^3B^4 A^4B^6	A^3B^2 A^4B^6	A^3B^6 A^4B^2	A^nB	A^nB^5	A^nB^3	A^nB^7

C	1	2	3	4	5	6	7	8	9	10	
#	1	1	1	1	2	2	2	2	2	2	$\mathcal{K}$
	1	1	1	1	1	1	1	1	1	1	G
	1	1	1	1	1	1	1	1	1	1	1
	1	1	–1	–1	1	1	–1	–1	1	1	2
	1	1	–1	–1	1	1	–1	–1	1	1	2
	1	–1	i	–i	1	–1	i	–i	1	–1	3
	1	–1	–i	i	1	–1	–i	i	1	–1	3
	1	–1	i	–i	1	–1	i	–i	1	–1	3
	1	–1	–i	i	1	–1	–i	i	1	–1	3
	2	2	2	2	c_1	c_1	c_1	c_1	c_2	c_2	4
	2	2	2	2	c_2	c_2	c_2	c_2	c_4	c_4	4
	2	2	2	2	c_4	c_4	c_4	c_4	c_1	c_1	4
	2	2	–2	–2	c_1	c_1	$-c_1$	$-c_1$	c_2	c_2	5
	2	2	–2	–2	c_2	c_2	$-c_2$	$-c_2$	c_4	c_4	5
	2	2	–2	–2	c_4	c_4	$-c_4$	$-c_4$	c_1	c_1	5
	2	–2	2i	–2i	c_1	$-c_1$	c_1i	$-c_1$i	c_2	$-c_2$	0
	2	–2	2i	–2i	c_2	$-c_2$	c_2i	$-c_2$i	c_4	$-c_4$	0
	2	–2	2i	–2i	c_4	$-c_4$	c_4i	$-c_4$i	c_1	$-c_1$	0
	2	–2	–2i	2i	c_1	$-c_1$	$-c_1$i	c_1i	c_2	$-c_2$	0
	2	–2	–2i	2i	c_2	$-c_2$	$-c_2$i	c_2i	c_4	$-c_4$	0
	2	–2	–2i	2i	c_4	$-c_4$	$-c_4$i	c_4i	c_1	$-c_1$	0
^	1	2	4	4	7	14	28	28	7	14	

$c_k = 2\cos(2k\pi/7)$, $\theta = e^{2\pi i/8} = \dfrac{1+i}{\sqrt{2}}$

C #	11 2	12 2	13 2	14 2	15 2	16 2	17 7	18 7	19 7	20 7	ℵ
	1	1	1	1	1	1	1	1	1	1	G
	1	1	1	1	1	1	–1	–1	–1	–1	1
	–1	–1	1	1	–1	–1	i	i	–i	–i	2
	–1	–1	1	1	–1	–1	–i	–i	i	i	2
	i	–i	1	–1	i	–i	θ	θ^5	θ^3	θ^7	3
	–i	i	1	–1	–i	i	θ^3	θ^7	θ	θ^5	3
	i	–i	1	–1	i	–i	θ^5	θ	θ^7	θ^3	3
	–i	i	1	–1	–i	i	θ^7	θ^3	θ^5	θ	3
	c_2	c_2	c_4	c_4	c_4	c_4	0	0	0	0	4
	c_4	c_4	c_1	c_1	c_1	c_1	0	0	0	0	4
	c_1	c_1	c_2	c_2	c_2	c_2	0	0	0	0	4
	$-c_2$	$-c_2$	c_4	c_4	$-c_4$	c_4	0	0	0	0	5
	$-c_4$	$-c_4$	c_1	c_1	$-c_1$	c_1	0	0	0	0	5
	$-c_1$	$-c_1$	c_2	c_2	$-c_2$	c_2	0	0	0	0	5
	c_2i	$-c_2i$	c_4	$-c_4$	c_4i	$-c_4i$	0	0	0	0	0
	c_4i	$-c_4i$	c_1	$-c_1$	c_1i	$-c_1i$	0	0	0	0	0
	c_1i	$-c_1i$	c_2	$-c_2$	c_2i	$-c_2i$	0	0	0	0	0
	$-c_2i$	c_2i	c_4	$-c_4$	$-c_4i$	c_4i	0	0	0	0	0
	$-c_4i$	c_4i	c_1	$-c_1$	$-c_1i$	c_1i	0	0	0	0	0
	$-c_1i$	c_1i	c_2	$-c_2$	$-c_2i$	c_2i	0	0	0	0	0
^	**28**	**28**	**7**	**14**	**28**	**28**	**8**	**8**	**8**	**8**	

	Classes	H	G/H	∩	
1	1+2+3+4+5+6+7+8+9+10+11+12+13+14+15+16	**28**	**2**	1	
2	1 + 2 + 5 + 6 + 9 + 10 + 13 + 14	**14**	**4**	2	
3	1 + 5 + 9 + 13	**7**	**8**	3	**G′**
4	1 + 2 + 3 + 4	**4**	$\mathbf{D_{14}}$	4	**Z**
5	1 + 2	**2**	$\mathbf{D_{7,4}}$	5	

Sylow subgroups: [**8**] × 7, [**7**] **Maximal subgroups:** [**28**], [**8**] × 7

56.10 = ⟨A^2, B^7, C^2, D^2, $DB = B^{-1}D$, $DC = ACD$⟩

1	2	3	4	5	6	7	8	9	10	11
1	A	B	AB	B^2 B^5	AB^2 AB^5	B^3 B^4	AB^3 AB^4	C AC	BC AB^6C	B^6C ABC

12	13	14	15	16	17
B^2C AB^5C	B^5C AB^2C	B^4C AB^3C	B^3C AB^4C	A^mB^nD	A^mB^nCD

C	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	
#	1	1	2	2	2	2	2	2	2	2	2	2	2	2	2	14	14	ℵ
	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	G
	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	–1	–1	1
	1	1	1	1	1	1	1	1	–1	–1	–1	–1	–1	–1	–1	1	–1	2
	1	1	1	1	1	1	1	1	–1	–1	–1	–1	–1	–1	–1	–1	1	3
	2	2	c_1	c_1	c_2	c_2	c_4	c_4	2	c_1	c_1	c_2	c_2	c_4	c_4	0	0	4
	2	2	c_2	c_2	c_4	c_4	c_1	c_1	2	c_2	c_2	c_4	c_4	c_1	c_1	0	0	4
	2	2	c_4	c_4	c_1	c_1	c_2	c_2	2	c_4	c_4	c_1	c_1	c_2	c_2	0	0	4
	2	2	c_1	c_1	c_2	c_2	c_4	c_4	–2	$-c_1$	$-c_1$	$-c_2$	$-c_2$	$-c_4$	$-c_4$	0	0	5
	2	2	c_2	c_2	c_4	c_4	c_1	c_1	–2	$-c_2$	$-c_2$	$-c_4$	$-c_4$	$-c_1$	$-c_1$	0	0	5
	2	2	c_4	c_4	c_1	c_1	c_2	c_2	–2	$-c_4$	$-c_4$	$-c_1$	$-c_1$	$-c_2$	$-c_2$	0	0	5
	2	–2	2	–2	2	–2	2	–2	0	0	0	0	0	0	0	0	0	6
	2	–2	c_1	$-c_1$	c_2	$-c_2$	c_4	$-c_4$	0	s_1i	s_{1i}	s_2i	s_2i	s_4i	s_4i	0	0	0
	2	–2	c_2	$-c_2$	c_4	$-c_4$	c_1	$-c_1$	0	s_2i	s_2i	s_4i	s_4i	s_1i	s_1i	0	0	0
	2	–2	c_4	$-c_4$	c_1	$-c_1$	c_2	$-c_2$	0	s_4i	s_4i	s_1i	s_1i	s_2i	s_2i	0	0	0
	2	–2	c_1	$-c_1$	c_2	$-c_2$	c_4	$-c_4$	0	$-s_1i$	s_1i	$-s_2i$	s_2i	$-s_4i$	s_4i	0	0	0
	2	–2	c_2	$-c_2$	c_4	$-c_4$	c_1	$-c_1$	0	$-s_2i$	s_2i	$-s_4i$	s_4i	$-s_1i$	s_1i	0	0	0
	2	–2	c_4	$-c_4$	c_1	$-c_1$	c_2	$-c_2$	0	$-s_4i$	s_4i	$-s_1i$	s_1i	$-s_2i$	s_2i	0	0	0
^	1	2	7	14	7	14	7	14	2	14	14	14	14	14	14	2	4	

$c_k = 2\cos(2k\pi/7)$, $s_k = 2\sin(2\pi/7)$

	Classes	H	G/H	∩	
1	1+2+3+4+5+6+7+8+9+10+11+12+13+14+15	**$D_{14} \times 2$**	**2**	1	
2	1 + 2 + 3 + 4 + 5 + 6 + 7 + 8 + 16	**D_{28}**	**2**	2	
3	1 + 2 + 3 + 4 + 5 + 6 + 7 + 8 + 17	**$D_{7,4}$**	**2**	3	
4	1 + 2 + 9	**2^2**	**D_{14}**	4	
5	1 + 2	**2**	**$D_{14} \times 2$**	5	**Z**
6	1 + 3 + 5 + 7	**7**	**D_8**	6	
7	1 + 2 + 3 + 4 + 5 + 6 + 7 + 8	**14**	**2^2**	1∩2	**G′**

Sylow subgroups: [D_8] × 7, [7] **Maximal subgroups:** [**14 × 2**], [$D_{14} \times 2$], [$D_{7,4}$], [D_8] × 7

56.11 = ⟨A^2, B^7, $C^2 = A$, D^2, $[B,D] = B^5$, $[C,D] = A$⟩

1	2	3	4	5	6	7	8	9	10	11
1	A^2	B B^3	A^2B A^2B^6	B^2 B^5	A^2B^2 A^2B^5	B^3 B^4	A^2B^3 A^2B^4	A A^3	AB A^3B^6	AB^6 A^3B

12	13	14	15	16	17
AB^2 A^3B^5	AB^5 A^3B^2	AB^3 A^3B^4	AB^4 A^3B^3	B^nC A^2B^nC	AB^nC $A^3\,B^nC$

C	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	
#	1	1	2	2	2	2	2	2	2	2	2	2	2	2	2	14	14	ℵ
	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	G
	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	–1	–1	1
	1	1	1	1	1	1	1	1	–1	–1	–1	–1	–1	–1	–1	1	–1	2
	1	1	1	1	1	1	1	1	–1	–1	–1	–1	–1	–1	–1	–1	1	3
	2	2	c_1	c_1	c_2	c_2	c_4	c_4	2	c_1	c_1	c_2	c_2	c_4	c_4	0	0	4
	2	2	c_2	c_2	c_4	c_4	c_1	c_1	2	c_2	c_2	c_4	c_4	c_1	c_1	0	0	4
	2	2	c_4	c_4	c_1	c_1	c_2	c_2	2	c_4	c_4	c_1	c_1	c_2	c_2	0	0	4
	2	2	c_1	c_1	c_2	c_2	c_4	c_4	–2	$-c_1$	$-c_1$	$-c_2$	$-c_2$	$-c_4$	$-c_4$	0	0	5
	2	2	c_2	c_2	c_4	c_4	c_1	c_1	–2	$-c_2$	$-c_2$	$-c_4$	$-c_4$	$-c_1$	$-c_1$	0	0	5
	2	2	c_4	c_4	c_1	c_1	c_2	c_2	–2	$-c_4$	$-c_4$	$-c_1$	$-c_1$	$-c_2$	$-c_2$	0	0	5
	2	–2	2	–2	2	–2	2	–2	0	0	0	0	0	0	0	0	0	6
	2	–2	c_1	$-c_1$	c_2	$-c_2$	c_4	$-c_4$	0	s_1	s_1	s_2	s_2	s_4	s_4	0	0	0
	2	–2	c_2	$-c_2$	c_4	$-c_4$	c_1	$-c_1$	0	s_2	s_2	s_4	s_4	s_1	s_1	0	0	0
	2	–2	c_4	$-c_4$	c_1	$-c_1$	c_2	$-c_2$	0	s_4	s_4	s_1	s_1	s_2	s_2	0	0	0
	2	–2	c_1	$-c_1$	c_2	$-c_2$	c_4	$-c_4$	0	$-s_1$	s_1	$-s_2$	s_2	$-s_4$	s_4	0	0	0
	2	–2	c_2	$-c_2$	c_4	$-c_4$	c_1	$-c_1$	0	$-s_2$	s_2	$-s_4$	s_4	$-s_1$	s_1	0	0	0
	2	–2	c_4	$-c_4$	c_1	$-c_1$	c_2	$-c_2$	0	$-s_4$	s_4	$-s_1$	s_1	$-s_2$	s_2	0	0	0
^	1	2	7	14	7	14	7	14	4	28	28	28	28	28	28	2	2	

$c_k = 2\cos(2k\pi/7)$, $s_k = 2\sin(2\pi/7)$

	Classes	H	G/H	∩	
1	1+2+3+4+5+6+7+8+9+10+11+12+13+14+15	**$D_{14} \times 2$**	**2**	1	
2	1 + 2 + 3 + 4 + 5 + 6 + 7 + 8 + 16	**D_{28}**	**2**	2	
3	1 + 2 + 3 + 4 + 5 + 6 + 7 + 8 + 17	**$D_{7,4}$**	**2**	3	
4	1 + 2 + 9	**2^2**	**D_{14}**	4	
5	1 + 2	**2**	**$D_{14} \times 2$**	5	**Z**
6	1 + 3 + 5 + 7	**7**	**D_8**	6	
7	1 + 2 + 3 + 4 + 5 + 6 + 7 + 8	**14**	**2^2**	1∩2	**G′**

Sylow subgroups: [D_8] × 7, [7] **Maximal subgroups:** [28], [$D_{14} \times 2$] × 2, [D_8] × 7

56.12 = ⟨A^4, B^7, $C^2 = A^2$, $CA = A^{-1}C$, $CB = B^{-1}C$⟩

1	2	3	4	5	6	7	8	9	10	11
1	A^2	B B^3	A^2B A^2B^6	B^2 B^5	A^2B^2 A^2B^5	B^3 B^4	A^2B^3 A^2B^4	A A^3	AB A^3B^6	AB^6 A^3B

12	13	14	15	16	17
AB^2 A^3B^5	AB^5 A^3B^2	AB^3 A^3B^4	AB^4 A^3B^3	B^nC A^2B^nC	AB^nC A^3B^nC

C	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	
#	1	1	2	2	2	2	2	2	2	2	2	2	2	2	2	14	14	ℵ
	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	G
	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	–1	–1	1
	1	1	1	1	1	1	1	1	–1	–1	–1	–1	–1	–1	–1	1	–1	2
	1	1	1	1	1	1	1	1	–1	–1	–1	–1	–1	–1	–1	–1	1	3
	2	2	c_1	c_1	c_2	c_2	c_4	c_4	2	c_1	c_1	c_2	c_2	c_4	c_4	0	0	4
	2	2	c_2	c_2	c_4	c_4	c_1	c_1	2	c_2	c_2	c_4	c_4	c_1	c_1	0	0	4
	2	2	c_4	c_4	c_1	c_1	c_2	c_2	2	c_4	c_4	c_1	c_1	c_2	c_2	0	0	4
	2	2	c_1	c_1	c_2	c_2	c_4	c_4	–2	$-c_1$	$-c_1$	$-c_2$	$-c_2$	$-c_4$	$-c_4$	0	0	5
	2	2	c_2	c_2	c_4	c_4	c_1	c_1	–2	$-c_2$	$-c_2$	$-c_4$	$-c_4$	$-c_1$	$-c_1$	0	0	5
	2	2	c_4	c_4	c_1	c_1	c_2	c_2	–2	$-c_4$	$-c_4$	$-c_1$	$-c_1$	$-c_2$	$-c_2$	0	0	5
	2	–2	2	–2	2	–2	2	–2	0	0	0	0	0	0	0	0	0	6
	2	–2	c_1	$-c_1$	c_2	$-c_2$	c_4	$-c_4$	0	s_1	s_1	s_2	s_2	s_4	s_4	0	0	0
	2	–2	c_2	$-c_2$	c_4	$-c_4$	c_1	$-c_1$	0	s_2	s_2	s_4	s_4	s_1	s_1	0	0	0
	2	–2	c_4	$-c_4$	c_1	$-c_1$	c_2	$-c_2$	0	s_4	s_4	s_1	s_1	s_2	s_2	0	0	0
	2	–2	c_1	$-c_1$	c_2	$-c_2$	c_4	$-c_4$	0	$-s_1$	s_1	$-s_2$	s_2	$-s_4$	s_4	0	0	0
	2	–2	c_2	$-c_2$	c_4	$-c_4$	c_1	$-c_1$	0	$-s_2$	s_2	$-s_4$	s_4	$-s_1$	s_1	0	0	0
	2	–2	c_4	$-c_4$	c_1	$-c_1$	c_2	$-c_2$	0	$-s_4$	s_4	$-s_1$	s_1	$-s_2$	s_2	0	0	0
^	1	2	7	14	7	14	7	14	4	28	28	28	28	28	28	4	4	

$c_k = 2\cos(2k\pi/7)$, $s_k = 2\sin(2k\pi/7)$

	Classes	H	G/H	∩	
1	1+2+3+4+5+6+7+8+9+10+11+12+13+14+15	**$D_{14} \times 2$**	**2**	1	
2	1 + 2 + 3 + 4 + 5 + 6 + 7 + 8 + 16	**D_{28}**	**2**	2	
3	1 + 2 + 3 + 4 + 5 + 6 + 7 + 8 + 17	**$D_{7,4}$**	**2**	3	
4	1 + 2 + 9	**2^2**	**D_{14}**	4	
5	1 + 2	**2**	**$D_{14} \times 2$**	5	Z
6	1 + 3 + 5 + 7	**7**	**D_8**	6	
7	1 + 2 + 3 + 4 + 5 + 6 + 7 + 8	**14**	**2^2**		G′

Sylow subgroups: [Q_8] × 7, [7] **Maximal subgroups:** [28], [$D_{7,4}$] × 2, [Q_8] × 7

56.13 = ⟨A², B², C², D⁷, DA = ACD, DB = AD, DC = BD⟩

1	2			3	4
1	A	A^mB	A^mC	A^mBC	$A^mB^nC^rD$	$A^mB^nC^rD^2$

5	6	7	8
$A^mB^nC^rD^3$	$A^mB^nC^rD^4$	$A^mB^nC^rD^5$	$A^mB^nC^rD^6$

C	1	2	3	4	5	6	7	8	
#	1	7	8	8	8	8	8	8	$\mathfrak{K}$
	1	1	1	1	1	1	1	1	G
	1	1	θ	θ^2	θ^3	θ^4	θ^5	θ^6	1
	1	1	θ^2	θ^4	θ^6	θ	θ^3	θ^5	1
	1	1	θ^3	θ^6	θ^2	θ^5	θ	θ^4	1
	1	1	θ^4	θ	θ^5	θ^2	θ^6	θ^3	1
	1	1	θ^5	θ^3	θ	θ^6	θ^4	θ^2	1
	1	1	θ^6	θ^5	θ^4	θ^3	θ^2	θ	1
	7	–1	0	0	0	0	0	0	0
^	1	2	7	7	7	7	7	7	

$\theta = e^{2\pi i/7}$

	Classes	H	G/H	∩
1	1 + 2	2^3	7	1

Sylow subgroups: $[2^3]$, $[7] \times 8$ **Maximal subgroups:** $[2^3]$, $[7] \times 4$

57 = ⟨A⁵⁷⟩ = 19 × 3

57.2 = $M_{19,3}^{(7)}$ = ⟨A^{19}, B^3, $BA = A^7B$⟩

1	**2**	**3**	**4**	**5**
1	A A^7 A^{11}	A^2 A^3 A^{13}	A^4 A^6 A^9	A^5 A^{16} A^{17}

6	**7**	**8**	**9**
A^8 A^{12} A^{18}	A^{10} A^{13} A^{15}	A^nB	A^nB^2

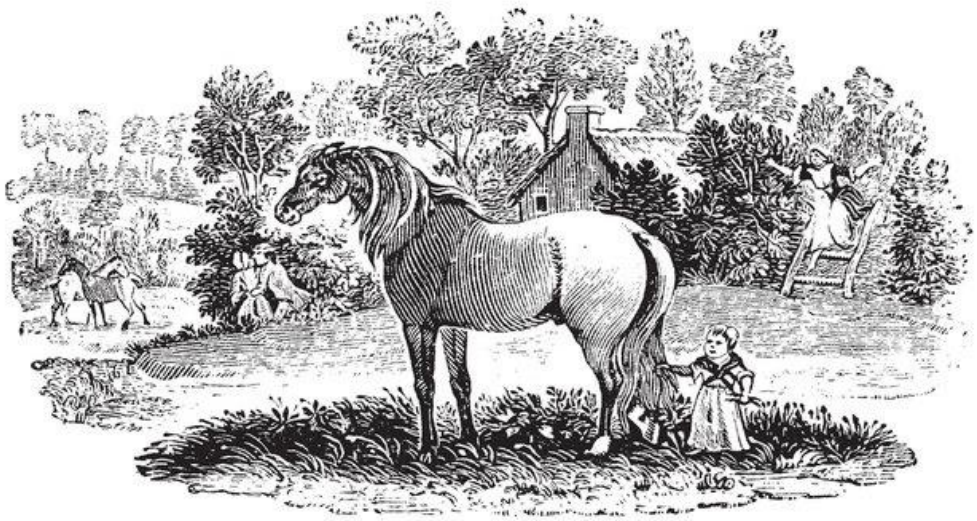
C	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	
#	**1**	**3**	**3**	**3**	**3**	**3**	**3**	**19**	**19**	ℵ
	1	1	1	1	1	1	1	1	1	G
	1	1	1	1	1	1	1	ω	ω^2	1
	1	1	1	1	1	1	1	ω^2	ω	1
	3	a	b	c	d	e	f	0	0	0
	3	b	c	e	f	d	e	0	0	0
	3	c	e	e	e	c	b	0	0	0
	3	d	f	e	f	b	e	0	0	0
	3	e	d	b	b	a	c	0	0	0
	3	f	e	b	e	c	b	0	0	0
^	**1**	**19**	**19**	**19**	**19**	**19**	**19**	**3**	**3**	

a = θ + θ^7 + θ^{11}, b = θ^2 + θ^3 + θ^{14}, c = θ^4 + θ^6 + θ^9, d = θ^5 + θ^{16} + θ^{17}, e = θ^8 + θ^{12} + θ^{18}, f = θ^{10} + θ^{13} + θ^{15} where θ = $e^{2\pi/19}$

	Classes	**H**	**G/H**	∩	
1	1 + 2 + 3 + 4 + 5 + 6 + 7	**19**	**3**	1	**G′**

Sylow and maximal subgroups: [**3**] × 19, [**19**]

58 = ⟨A^{58}⟩ = 29 × 2



58.2 = D_{58} = $\langle A^{29}, B^2, BA = A^{-1}B \rangle$

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
1	A A^{28}	A^2 A^{27}	A^3 A^{26}	A^4 A^{25}	A^5 A^{24}	A^6 A^{23}	A^7 A^{22}	A^8 A^{21}	A^9 A^{20}	A^{10} A^{19}	A^{11} A^{18}	A^{12} A^{17}	A^{13} A^{16}	A^{14} A^{15}	A^nB

C	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	
#	1	2	2	2	2	2	2	2	2	2	2	2	2	2	2	29	$\mathfrak{K}$
1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	G
1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	–1	1
2	c_1	c_2	c_3	c_4	c_5	c_6	c_7	c_8	c_9	c_{10}	c_{11}	c_{12}	c_{13}	c_{14}	0	0	
2	c_2	c_4	c_6	c_8	c_{10}	c_{12}	c_{14}	c_{13}	c_{11}	c_9	c_7	c_5	c_3	c_1	0	0	
2	c_3	c_6	c_9	c_{12}	c_{14}	c_{11}	c_8	c_5	c_2	c_1	c_4	c_7	c_{10}	c_{13}	0	0	
2	c_4	c_8	c_{12}	c_{13}	c_9	c_5	c_1	c_4	c_7	c_{11}	c_{15}	c_{10}	c_{16}	c_2	0	0	
2	c_5	c_{10}	c_{14}	c_9	c_4	c_1	c_6	c_{11}	c_{13}	c_8	c_3	c_2	c_7	c_{12}	0	0	
2	c_6	c_{12}	c_{11}	c_5	c_1	c_7	c_{13}	c_{10}	c_4	c_2	c_8	c_{14}	c_9	c_3	0	0	
2	c_7	c_{14}	c_8	c_1	c_6	c_{13}	c_9	c_2	c_5	c_{12}	c_{10}	c_3	c_4	c_{11}	0	0	
2	c_8	c_{13}	c_5	c_4	c_{11}	c_{10}	c_2	c_6	c_{14}	c_7	c_1	c_9	c_{12}	c_4	0	0	
2	c_9	c_{11}	c_2	c_7	c_{13}	c_4	c_5	c_{14}	c_6	c_3	c_{12}	c_8	c_1	c_{10}	0	0	
2	c_{10}	c_9	c_1	c_{11}	c_8	c_2	c_{12}	c_7	c_3	c_{13}	c_6	c_4	c_{14}	c_5	0	0	
2	c_{11}	c_7	c_4	c_{15}	c_3	c_8	c_{10}	c_1	c_{12}	c_6	c_5	c_{13}	c_2	c_9	0	0	
2	c_{12}	c_5	c_7	c_{10}	c_2	c_{14}	c_3	c_9	c_8	c_4	c_{13}	c_1	c_{11}	c_6	0	0	
2	c_{13}	c_3	c_{10}	c_{16}	c_7	c_9	c_4	c_{12}	c_1	c_{14}	c_2	c_{11}	c_5	c_8	0	0	
2	c_{14}	c_1	c_{13}	c_2	c_{12}	c_3	c_{11}	c_4	c_{10}	c_5	c_9	c_6	c_8	c_7	0	0	
^	1	29	29	29	29	29	29	29	29	29	29	29	29	29	29	2	

$c_k = 2\cos(2k\pi/29)$

	Classes	H	G/H	∩	
1	1 + 2 + 3 + 4 + 5 + 6 + 7 + 8 + 9 + 10 + 11 + 12 + 13 + 14 + 15	**29**	**2**	1	**G′**

Sylow and maximal subgroups: [**2**] × 29, [**29**]

59 = $\langle A^{59} \rangle$

60 = $\langle A^{60} \rangle$ = 15 × 4

60.02 = 30 × 2 = $\langle A^{30}, B^2 \rangle$

60.03 = D_6 × 10 = $\langle A^3, B^2, C^{10}, BA = A^{-1}B \rangle$

60.04 = $D_{3,4}$ × 5 = $\langle A^3, B^4, C^5, BA = A^{-1}B \rangle$

60.05 = A_4 × 5 = $\langle A^2, B^2, C^3, D^5, CA = ABC, CB = AC \rangle$

60.06 = $D_{10} \times 6 = \langle A^5, B^2, C^6, BA = A^{-1}B \rangle$

60.07 = $D_{5,4} \times 3 = \langle A^5, B^4, C^3, BA = A^{-1}B \rangle$

60.08 = $M_{4,5}^{(2)} \times 3 = \langle A^4, B^5, C^3, BA = A^3B \rangle$

60.09 = $D_{30} \times 2 = \langle A^{15}, B^2, C^2, BA = A^{-1}B \rangle$

60.10 = $D_{15,4} = \langle A^{15}, B^4, BA = A^{-1}B \rangle$

1	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**
1	B^2	A^5 A^{10}	A^5B^2 $A^{10}B^2$	A^3 A^{12}	A^6 A^9	A A^{14}	A^2 A^{13}	A^4 A^{11}	A^7 A^8

11	**12**	**13**	**14**	**15**	**16**	**17**	**18**
A^3B^2 A^9B^2	A^6B^2 A^9B^2	AB^2 $A^{14}B^2$	A^2B^2 $A^{13}B^2$	A^4B^2 $A^{11}B^2$	A^7B^2 A^8B^2	A^nB	A^nB^3

C	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**	**13**	**14**	**15**	**16**	**17**	**18**	
#	**1**	**1**	**2**	**2**	**2**	**2**	**2**	**2**	**2**	**2**	**2**	**2**	**2**	**2**	**2**	**2**	**15**	**15**	$\mathfrak{K}$
	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	G
	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	–1	–1	1
	1	–1	1	–1	1	1	1	1	1	1	–1	–1	–1	–1	–1	–1	i	–i	2
	1	–1	1	–1	1	1	1	1	1	1	–1	–1	–1	–1	–1	–1	–i	i	2
	2	2	–1	–1	2	2	–1	–1	–1	–1	2	2	–1	–1	–1	–1	0	0	3
	2	–2	–1	1	2	2	–1	–1	–1	–1	–2	–2	1	1	1	1	0	0	4
	2	–2	–1	1	c_3	c_6	c_1	c_2	c_4	c_7	$-c_3$	$-c_6$	$-c_1$	$-c_2$	$-c_4$	$-c_7$	0	0	0
	2	–2	–1	1	c_6	c_3	c_2	c_4	c_7	c_1	$-c_6$	$-c_3$	$-c_2$	$-c_4$	$-c_7$	$-c_1$	0	0	0
	2	–2	–1	1	c_3	c_6	c_4	c_7	c_1	c_2	$-c_3$	$-c_6$	$-c_4$	$-c_7$	$-c_1$	$-c_2$	0	0	0
	2	–2	–1	1	c_6	c_3	c_7	c_1	c_2	c_4	$-c_6$	$-c_3$	$-c_7$	$-c_1$	$-c_2$	$-c_4$	0	0	0
	2	2	–1	–1	c_3	c_6	c_1	c_2	c_4	c_7	c_3	c_6	c_1	c_2	c_4	c_7	0	0	5
	2	2	–1	–1	c_6	c_3	c_2	c_4	c_7	c_1	c_6	c_3	c_2	c_4	c_7	c_1	0	0	5
	2	2	–1	–1	c_3	c_6	c_4	c_7	c_1	c_2	c_3	c_6	c_4	c_7	c_1	c_2	0	0	5
	2	2	–1	–1	c_6	c_3	c_7	c_1	c_2	c_4	c_6	c_3	c_7	c_1	c_2	c_4	0	0	5
	2	2	2	2	c_6	c_3	c_3	c_6	c_3	c_6	c_6	c_3	c_3	c_6	c_3	c_6	0	0	6
	2	2	2	2	c_3	c_6	c_6	c_3	c_6	c_3	c_3	c_6	c_6	c_3	c_6	c_3	0	0	6
	2	–2	2	–2	c_6	c_3	c_3	c_6	c_3	c_6	$-c_6$	$-c_3$	$-c_3$	$-c_6$	$-c_3$	$-c_6$	0	0	7
	2	–2	2	–2	c_3	c_6	c_6	c_3	c_6	c_3	$-c_3$	$-c_6$	$-c_6$	$-c_3$	$-c_6$	$-c_3$	0	0	7
^	**1**	**2**	**3**	**6**	**5**	**5**	**15**	**15**	**15**	**15**	**10**	**10**	**30**	**30**	**30**	**30**	**4**	**4**	

$c_k = 2\cos(2k\pi/15)$

	Classes	H	G/H	∩	
1	1+2+3+4+5+6+7+8+9+10+11+12+13+14+15+16	**30**	**2**	1	
2	1 + 3 + 5 + 6 + 7 + 8 + 9 + 10	**15**	**4**	2	**G′**
3	1 + 2 + 5 + 6 + 11 + 12	**10**	**D_6**	3	
4	1 + 5 + 6	**5**	**$D_{3,4}$**	4	
5	1 + 2	**2**	**D_{30}**	5	**Z**
6	1 +2 + 3 + 4	**6**	**D_{10}**	6	
7	1 + 3	**3**	**$D_{5,4}$**	7	

Sylow subgroups: [**4**] × 15, [**3**], [**5**] **Maximal subgroups:** [**30**], [$\mathbf{D_{5,4}}$] × 3, [$\mathbf{D_{3,4}}$] × 5

60.11 = $D_{10} \times D_6 = \langle A^5, B^2, C^3, D^2, BA = A^{-1}B, DC = C^{-1}D \rangle$

60.12 = $\langle A^5, B^3, C^4, CA = A^3C, CB = B^{-1}C \rangle$

1	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**
1	B B^2	A A^2 A^3 A^4	AB	A^2B	C^2	BC^2	A^mB^n C	C^3

C	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	
#	**1**	**2**	**4**	**4**	**4**	**5**	**10**	**15**	**15**	ℋ
	1	1	1	1	1	1	1	1	1	G
	1	1	1	1	1	1	1	–1	–1	1
	1	1	1	1	1	–1	–1	i	–i	2
	1	1	1	1	1	–1	–1	–i	i	2
	2	–1	2	–1	–1	2	–1	0	0	3
	2	–1	2	–1	–1	–2	1	0	0	4
	4	4	–1	–1	–1	0	0	0	0	5
	4	–2	–1	$\frac{1+\sqrt{15}}{2}$	$\frac{1-\sqrt{15}}{2}$	0	0	0	0	0
	4	–2	–1	$\frac{1-\sqrt{15}}{2}$	$\frac{1+\sqrt{15}}{2}$	0	0	0	0	0
^	**1**	**3**	**5**	**15**	**15**	**2**	**6**	**4**	**4**	

	Classes	H	G/H	∩	
1	1 + 2 + 3 + 4 + 5 + 6 + 7	**$D_{10} \times 3$**	**2**	1	**G′**
2	1 + 2 + 3 + 4 + 5			2	
3	1 + 3 + 6	**D_{10}**	**D_6**	3	
4	1 + 3	**5**	**$D_{3,4}$**	4	
5	1 + 2	**3**	**$M_{5,4}^{(3)}$**	5	

Sylow subgroups:
[**4**] × 15, [**3**], [**5**]

Maximal subgroups:
[$\mathbf{D_{10} \times 3}$], [$\mathbf{M_{5,4}^{(2)}}$] × 3, [$\mathbf{D_{3,4}}$] × 5

60.13 = A_5

1	2	3	4	5
1	(××)(××)	(×××)	(×××××)	(×××××)

C	1	2	3	4	5	
#	1	15	20	12	12	$\mathfrak{K}$
	1	1	1	1	1	G
	3	–1	0	$\frac{1+\sqrt{5}}{2}$	$\frac{1-\sqrt{5}}{2}$	0
	3	–1	0	$\frac{1-\sqrt{5}}{2}$	$\frac{1+\sqrt{5}}{2}$	0
	4	0	1	–1	–1	0
	5	–1	–1	0	0	0
^	1	2	3	5	5	

SIMPLE GROUP – No proper, non-trivial normal subgroup
Sylow subgroups: [2^2] × 5, [**3**] × 10, [**5**] × 6
Maximal subgroups: [A_4] × 5, [D_{10}] × 6, [D_6] × 10

61 = ⟨A^{61}⟩

62 = ⟨A^{62}⟩ = 31 × 2

62.2 = D_{62} = $\langle A^{31}, B^2, BA = A^{-1}B \rangle$

1	2	3	4	5	6	7	8	9	10
1	A A^{30}	A^2 A^{29}	A^3 A^{28}	A^4 A^{27}	A^5 A^{26}	A^6 A^{25}	A^7 A^{24}	A^8 A^{23}	A^9 A^{22}

11	12	13	14	15	16	17
A^{10} A^{21}	A^{11} A^{20}	A^{12} A^{19}	A^{13} A^{18}	A^{14} A^{17}	A^{15} A^{16}	A^nB

C	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	
#	1	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	31	ℋ
1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	G
1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	–1	1
2	c_1	c_2	c_3	c_4	c_5	c_6	c_7	c_8	c_9	c_{10}	c_{11}	c_{12}	c_{13}	c_{14}	c_{15}	i	0	
2	c_2	c_4	c_6	c_8	c_{10}	c_{12}	c_{14}	c_{15}	c_{13}	c_{11}	c_9	c_7	c_5	c_3	c_1	–i	0	
2	c_3	c_6	c_9	c_{12}	c_{15}	c_{13}	c_{10}	c_7	c_4	c_1	c_2	c_5	c_8	c_{11}	c_{14}	0	0	
2	c_4	c_8	c_{12}	c_{15}	c_{11}	c_7	c_3	c_1	c_5	c_9	c_{10}	c_{14}	c_{10}	c_6	c_2	0	0	
2	c_5	c_{10}	c_{15}	c_{11}	c_6	c_5	c_4	c_9	c_{14}	c_{12}	c_7	c_2	c_3	c_6	c_{13}	0	0	
2	c_6	c_{12}	c_{13}	c_7	c_5	c_5	c_{11}	c_{14}	c_8	c_2	c_4	c_{10}	c_{15}	c_9	c_3	0	0	
2	c_7	c_{14}	c_{10}	c_3	c_4	c_{11}	c_{13}	c_6	c_1	c_8	c_{15}	c_9	c_2	c_5	c_{12}	0	0	
2	c_8	c_{15}	c_7	c_1	c_9	c_{14}	c_6	c_2	c_{10}	c_{13}	c_5	c_3	c_{11}	c_{12}	c_4	0	0	
2	c_9	c_{13}	c_4	c_5	c_{14}	c_8	c_1	c_{10}	c_{12}	c_3	c_6	c_{15}	c_7	c_2	c_{11}	0	0	
2	c_{10}	c_{11}	c_1	c_9	c_{12}	c_2	c_8	c_{13}	c_3	c_7	c_{14}	c_4	c_6	c_{15}	c_5	0	0	
2	c_{11}	c_9	c_2	c_{10}	c_7	c_4	c_{15}	c_5	c_6	c_{14}	c_3	c_8	c_{12}	c_1	c_{10}	0	0	
2	c_{12}	c_7	c_5	c_{14}	c_2	c_{10}	c_9	c_3	c_{15}	c_{14}	c_8	c_{11}	c_1	c_{13}	c_6	0	0	
2	c_{13}	c_5	c_8	c_{10}	c_3	c_{15}	c_2	c_{11}	c_7	c_6	c_{12}	c_1	c_{14}	c_4	c_9	0	0	
2	c_{14}	c_3	c_{11}	c_6	c_6	c_9	c_5	c_{12}	c_2	c_{15}	c_1	c_{13}	c_4	c_{10}	c_7	0	0	
2	c_{15}	c_1	c_{14}	c_2	c_2	c_3	c_{12}	c_4	c_{11}	c_5	c_{10}	c_6	c_9	c_7	c_8	0	0	
^	1	31	31	31	31	31	31	31	31	31	31	31	31	31	31	31	2	

$c_k = 2\cos(2k\pi/31)$

Normal subgroups

	Classes	H	G/H	∩
1	1 + 2 + 3 + 4 + 5 + 6 + 7 + 8 + 9 + 10 + 11 + 12 + 13 + 14 + 15 + 16	**31**	**2**	1

Sylow and maximal subgroups: [**2**] × 31, [**31**]

63 = $\langle A^{63} \rangle$ = 9 × 7

63.2 = 21 × 3 = $\langle A^{21}, B^3 \rangle$

63.3 = $M_{7,3}^{(4)}$ × 3 = $\langle A^7, B^3, C^3, BA = A^4B \rangle$

63.4 = $M_{7,9}^{(4)} = \langle A^7, B^9, BA = A^4B \rangle$

1	2	3	4	5	6	7	8	9
1	B^3	B^6	A A^2 A^4	A^3 A^5 A^6	AB^3 A^2B^3 A^4B^3	A^3B^3 A^5B^3 A^6B^3	AB^6 A^2B^6 A^4B^6	A^3B^6 A^5B^6 A^6B^6

10	11	12	13	14	15
A^nB	A^nB^2	A^nB^4	A^nB^5	A^nB^7	A^nB^8

C	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	
#	1	1	1	3	3	3	3	3	3	7	7	7	7	7	7	ℵ
	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	G
	1	1	1	1	1	1	1	1	1	ω	$ω^2$	ω	$ω^2$	ω	$ω^2$	1
	1	1	1	1	1	1	1	1	1	$ω^2$	ω	$ω^2$	ω	$ω^2$	ω	1
	1	ω	$ω^2$	1	1	ω	ω	$ω^2$	$ω^2$	θ	$θ^2$	$θ^4$	$θ^5$	$θ^7$	$θ^8$	2
	1	$ω^2$	ω	1	1	$ω^2$	$ω^2$	ω	ω	$θ^2$	$θ^4$	$θ^8$	θ	$θ^5$	$θ^7$	2
	1	ω	$ω^2$	1	1	ω	ω	$ω^2$	$ω^2$	$θ^4$	$θ^8$	$θ^7$	$θ^2$	θ	$θ^5$	2
	1	$ω^2$	ω	1	1	$ω^2$	$ω^2$	ω	ω	$θ^5$	θ	$θ^2$	$θ^7$	$θ^8$	$θ^4$	2
	1	ω	$ω^2$	1	1	ω	ω	$ω^2$	$ω^2$	$θ^7$	$θ^5$	θ	$θ^8$	$θ^4$	$θ^2$	2
	1	$ω^2$	ω	1	1	$ω^2$	$ω^2$	ω	ω	$θ^8$	$θ^7$	$θ^5$	$θ^4$	$θ^2$	θ	2
	3	3	3	a	b	a	b	a	b	0	0	0	0	0	0	3
	3	3	3	b	a	b	a	b	a	0	0	0	0	0	0	3
	3	3ω	$3ω^2$	a	b	aω	bω	$aω^2$	$bω^2$	0	0	0	0	0	0	0
	3	$3ω^2$	3ω	a	b	$aω^2$	$bω^2$	aω	bω	0	0	0	0	0	0	0
	3	3ω	$3ω^2$	b	a	bω	aω	$bω^2$	$aω^2$	0	0	0	0	0	0	0
	3	$3ω^2$	3ω	b	a	$bω^2$	$aω^2$	bω	aω	0	0	0	0	0	0	0
^	1	3	3	7	7	21	21	21	21	9	9	9	9	9	9	

$$a = \frac{-1 + \sqrt{7}i}{2},\ b = \frac{-1 - \sqrt{7}i}{2},\ \omega = e^{2\pi/3}$$

	Classes	H	G/H	∩	
1	1 + 2 + 3 + 4 + 5 + 6 + 7 + 8 + 9	**21**	**3**	1	
2	1 + 4 + 5	**7**	**9**	2	**G′**
3	1 + 2 + 3	**3**	$\mathbf{M_{7,3}^{(4)}}$	3	

Sylow subgroups: [**9**] × 7, [**7**] **Maximal subgroups:** [**21**], [**9**] × 6